Title: Diving into Pool Geometry

Brief Overview:

This performance-based assessment integrates geometric problem solving with real-world application. This unit will involve the mathematical skills of investigating and measuring perimeter, area, and volume by using analytical problem solving; comparing the relationships between perimeter and area; using a compass to design a pattern; and drawing models to scale. The students will use all of these skills to complete a cumulative project in which they will design a pool that meets specific standards in their backyard, design and order the proper amount of tiles for decoration, and use both oral and written communication skills to discuss and explain their creations.

Links to Standards:

- **Mathematics as Problem Solving**
  Students will demonstrate their ability to solve problems in mathematics including problems with open-ended answers and problems which are solved in a cooperative atmosphere.

- **Mathematics as Communication**
  Students will demonstrate their ability to communicate mathematically. They will read, write, and discuss mathematics with language and the signs, symbols, and terms of the discipline.

- **Mathematics as Reasoning**
  Students will demonstrate their ability to reason mathematically. They will make conjectures, gather evidence, and build arguments.

- **Mathematical Connections**
  Students will demonstrate their ability to connect mathematics topics within the discipline and with other disciplines.

- **Estimation & Computation**
  Students will demonstrate their ability to apply estimation strategies in computation, with the use of technology, in measurement, and in problem solving. They will determine reasonableness of solutions.

- **Number Sense & Operations**
  Students will demonstrate their ability to describe and apply number relationships using concrete and abstract materials. They will choose appropriate operations and describe effects of operations on numbers.

- **Geometry & Spatial Sense**
  Students will demonstrate their ability to describe and apply geometric relationships using one, two, and three dimensional objects. They will demonstrate congruency, similarity, symmetry, and reflections and apply these concepts to the solution of geometric problems.

- **Measurement**
  Students will demonstrate and apply concepts of measurement using non-standard and standard units and metric and customary units. They will estimate and verify measurements. They will apply measurement to interdisciplinary and real-world problem solving situations.

- **Patterns & Relationships**
  Students will demonstrate their ability to recognize numeric and geometric relationships and will generalize a relationship from data.

Grade/Level:

Grades 3-5

Duration/Length:

This project will take approximately seven one-hour lessons without the extension activities.

Prerequisite Knowledge:

Students should have working knowledge of the following skills (for example):

- Measuring in metric units.
- Geometrical names and shapes of polygons.
- Computation of whole numbers.

Objectives:

Students will:

- work cooperatively with partners.
- use analytical problem solving in order to identify possible solutions for a real-life problem.
- measure the area and perimeter of polygons using standard and nonstandard units.
- use scaling to represent an object.
- compare relationships between area and perimeter.
- calculate the volume of space figures.
- use compasses to create a tile design.
- calculate total ordering prices for given items.
- use references in order to obtain information and make reasonable decisions.

Materials/Resources/Printed Materials:

Day 1:

- Transparency of the vignette, Resource Sheet #1
- Meter sticks, 1 per 4 students
- One Centimeter grid paper for students, Resource Sheet #2--2 sheets per student
- Construction paper 9” x 12” to glue yard grids
- Glue
- Homework Resource Sheet #3, today cut section for Day 1

Day 2:

- Non-standard units of measurement (teacher’s choice)
- Homework strip for Day 2
- Rulers
- Grid paper for homework

Day 3:

- Construction paper, 12 cm x 12 cm, for each student (for pool deck)
- Transparency of one centimeter grid paper, Resource Sheet #2
- Area Worksheet, Resource Sheet #4
- Centimeter cubes
- Homework strip for Day 3

Day 4:

- Centimeter cubes
- Grid paper
- Scissors and glue
- Homework strip of Day 4

Day 5:

- Centimeter cubes
- Homework strip for Day 5
- Space figure models or volume models for reference

Day 6:

- Compasses
- White paper, 20 cm x 20 cm
- Transparency of Resource Sheet #5, Compass Art
- Markers or crayons

Day 7:

- Diving Into Pool Geometry Data, Resource Sheet #6
- Student Checklist, Resource Sheet #7
- Scoring Rubric, Resource Sheet #8

Development/Procedures:

Day 1: Analyze Problem and Scaling

- Present the scenario to the students.
- Use vignette to determine the problem and list necessary knowledge to solve the problem.
- Go to a large area and measure the size of the yard using meter sticks.
- Discuss how to represent this area in the classroom; arrive at the conclusion that scaling will be necessary.
- Distribute grid paper which will represent the size of the yard, 17 m x 24 m. Use the scale model, $1\ m^2 = 1\ cm^2$. For a more advanced group, the entire activity may be modified to the scale of your choice.
- Complete journal reflection: Why is scaling a necessary skill in a problem like you've seen today? In what similar situations can you imagine the skill of scaling being beneficial?
- Distribute homework for Day 1 (see materials for explanation of Resource Sheet.).

Day 2: Perimeter

- Share homework assignments and elicit the definition of perimeter.
- In teams, measure how many non-standard units (teacher's choice) a particular object is. Compare them and discuss why the measurements vary. Stress the importance of standard units.
- Provide a variety of experiences measuring the perimeters of polygons. Generate a formula for finding the perimeter (p= sum of length of sides), including the formula used for parallelograms/rectangles/squares (p= 2l + 2w). The students should now be able to calculate the perimeter of the yard.
- Distribute homework for Day 2 along with grid paper for the assignment.

Day 3: Area

- Distribute a piece of construction paper (precut by teacher) to represent the pool deck (12 cm x 12 cm). Students share their designs in groups and discuss reasonableness of the pool shape.
- Demonstrate reasonable and unreasonable sample shapes on a grid transparency in order to elicit the idea that the pool needs to fit within a certain "area", while still leaving room for the deck.
- Define area and discuss other situations in which knowing area is useful. Count the grid blocks to find the area of the yard, the pool deck, and the pool. Provide students with multiple hands-on opportunities to find the area of a variety of polygons. Determine the formula and proper notation for area. (See Worksheet) (Depending on ability level, area of triangles may or may not be introduced.)
- Working in pairs, the students will design a pool that has an area of 60 m^2 and stays within the 12 cm x 12 cm deck perimeter. After designing, students will use both manipulatives and formulas in order to verify the accuracy of the area.
- Distribute homework assignment for Day 3.

Day 4: Relationships between Perimeter and Area

- Investigate the relationship between perimeter and area using a variety of different techniques. One activity is to distribute centimeter cubes to each pair. Students will build shapes using twelve cubes to find the shape with the greatest perimeter. The sides of the cubes must be in full contact. Students should record their work on centimeter grid paper and find area and perimeter for each shape.
- Discuss importance of maximum area in a given space with respect to the pool the students are designing. In pairs, students will design a final shape for their pool in which the area totals 60 m^2. Upon approval of final design, pools may be cut and glued onto pool deck and then onto grid yard in a location of the students' choice.
- Design 2 gardens that have an area of 12 m^2 and glue onto the grid yard in a location of the students' choice. Students may choose their favorite design from the previous activity.
- Distribute homework assignment for Day 4.

Day 5: Volume

- Share pool designs with class.
- Tell students they will be building a rectangular solid using 24 centimeter cubes. Discuss the characteristics of a rectangular solid while referring to their models, emphasizing length, height, width, and their relationships.
- Allow students to build a variety of rectangular prisms and record length, height, width, and number of cubes used.

- Define volume and examine recorded measurements. Have students discover the formula for volume.
- Have pairs find the volume of their pools. The depth of the pool is 2 meters. (Advanced group may have a shallow and deep end.) Share equations in teams to check accuracy.
- Discuss volume and consumer awareness before assigning homework.
- Distribute homework assignment for Day 5.

Day 6: Tiling and Compass Art

- Examine situations in order to determine which measurement skill is applicable using the resource sheet.
- Determine which measurement skill should be used to calculate the number of tiles needed to border your pool at the water line. Each tile is 20 cm x 20 cm; adjust measurements according to the level of your students. Calculate the number of tiles needed to border the perimeter of your pool.
- Distribute compasses and white paper 20 cm x 20 cm and use Resource Sheet #5, Compass Art, to design a tile. When students are finished, they may color their designs.

Day 7: Calculating Cost

- Share tile designs, then determine the cost of the tiles if each one costs $3.59. How much will it cost to complete the tiling of the pool? (For higher level students, calculate the tax for the order.)
- Determine the cost of the pool water if the price is $1.10 per ten liters of water, delivery included. See Language Arts extension for a prerequisite activity to this problem.
- Complete Diving Into Pool Geometry Data Sheet, Resource Sheet #6 and checklist, Resource Sheet #7. This writing and the final project can be shared with the whole class or displayed in the classroom or hall.

Performance Assessment:

Students can be evaluated based on the following:

- Participation and performance in pairs and in whole group discussions.
- Use of math language throughout the unit.
- Journal reflections.
- Homework assignments.
- Summative assessment of the project can be assessed with the scoring rubric, Resource Sheet #8.

Extension/Follow Up:

- Invitations to the opening day of the pool.
- Use phone book to identify pool water companies and call the companies to get actual costs for delivery of water.
- Use quotation mark notation to show a conversation on the phone with the pool company ordering water and other supplies
- Investigate shapes without 90 degree angles: find the area and volume of triangles, divide slanted polygons into rectangles and triangles to find area, perimeter, and volume, find the area of a circle.
- Landscape yard with trees, shrubs, and play equipment in areas where there is enough room.
- Design garden with various plants and vegetables, leaving adequate room for healthy plant growth.

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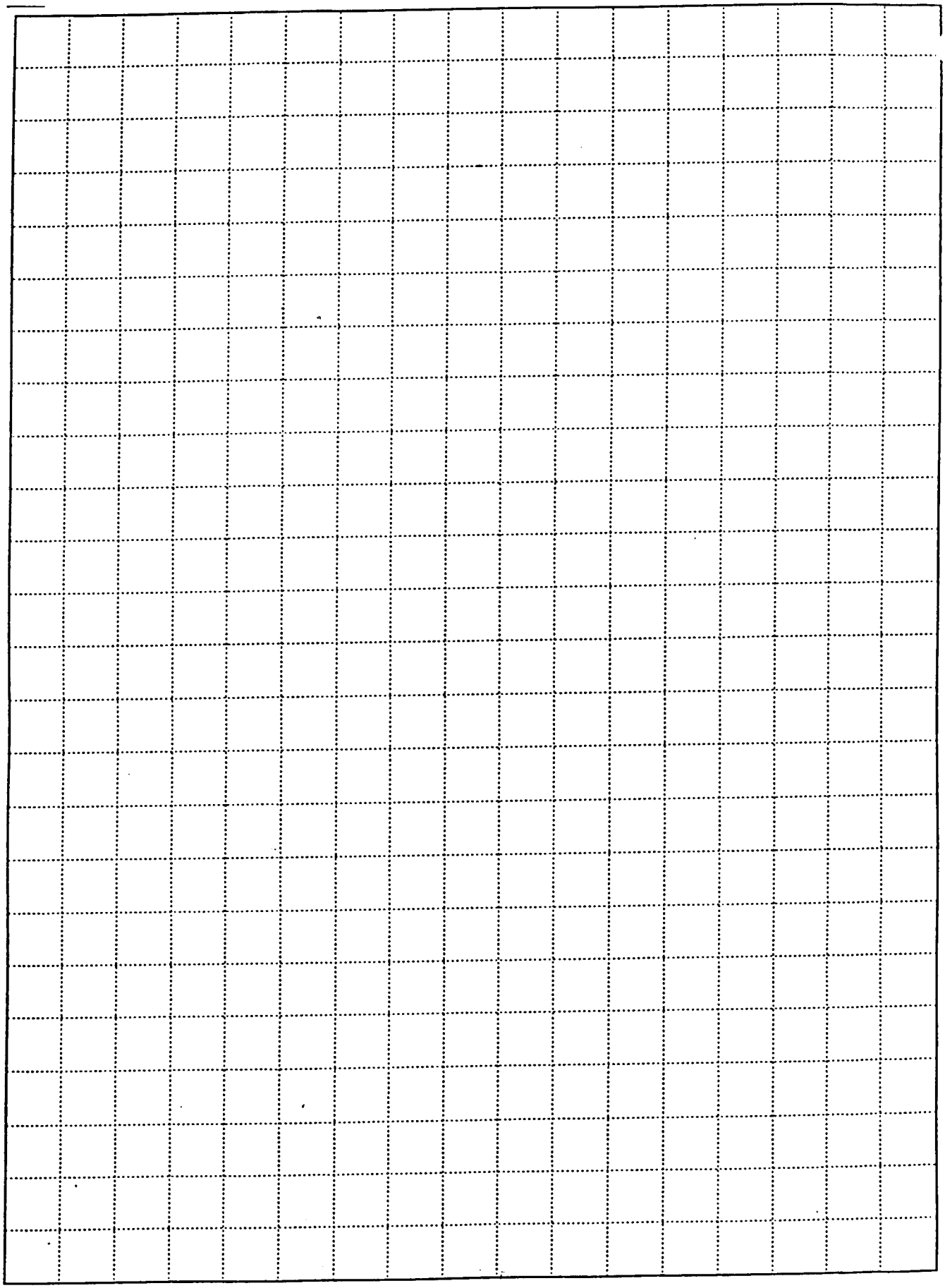
Diving into Pool Geometry Vignette

Your family is about to move into a new house with a backyard that needs a lot of landscaping. To your surprise and excitement, your family has decided to build a new inground pool in the backyard. They are also planning a beautiful vegetable garden. The only thing that they ask of you is to help with the pool design (and of course the testing and cleaning once it is built)!

Several guidelines apply to the pool design. Your backyard is 17 meters long and 24 meters wide. Your family's two gardens need to be 12 square meters each in area. The pool deck has four equal sides which measure 12 meters each. The pool can be any polygon shape using straight lines that form right angles but will have a total area of 60 square meters.

After designing the pool, you will also be responsible for designing and ordering the expensive tile that will go along the edge of the water. Then for the fun and crucial element--water! You will have to decide how much water must be delivered to your new pool before you take your first plunge!

Before we begin to design the pool, what steps do you see that we will need to learn to do in order to be successful and make this as inexpensive as possible? List your ideas below.



Day 1:
Measure the lengths of each of the walls of your bedroom using the width (short side) of a regular piece of notebook paper as your unit of measurement. Then, take the measurements and make each unit equal to a unit on your grid paper in order to draw your bedroom to scale so that you can share with the class tomorrow. Be ready to discuss the following questions:

- What did you encounter in scaling the measurement of your bedroom that you did not encounter when scaling the size of your yard in class today?
- What was the same and different about your drawings of the yard and your bedroom?

Day 2:
Estimate the length and width measurements of your closet and your bedroom. Use a ruler to measure the closet and room in inches. Find the perimeter of each. Tell why you would need to use perimeter in real life in 3-4 sentences.

Day 3:
Your family has already designed a pool deck that is a square shape. If one side is 12 meters in length, what is the perimeter of the pool deck? Use the grid paper in order to design at least 3 possible pool shapes as you can that will fit within this pool deck's perimeter. The pool's area should be 60 cm^2. Remember to leave room for the deck. Be prepared to share and discuss the reasonableness and variety of your pool designs with classmates tomorrow.

Day 4:
In your journal, write a paragraph using as many math terms as possible to describe your yard design. Be sure to proofread and edit. Try to use at least 15 math terms and underline them.

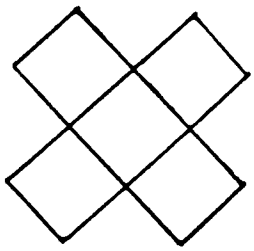
Day 5:
Find and record 5-10 situations in your home in which volume is labeled or would be useful to know.
What kinds of packages do you find to be the most attractive? What considerations will companies make when designing packages for their products?

Diving Into Geometry: Activity Worksheet for Area

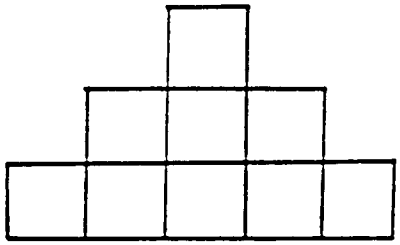
Area: The number of square units contained in a surface.

Find the area by counting the square units.

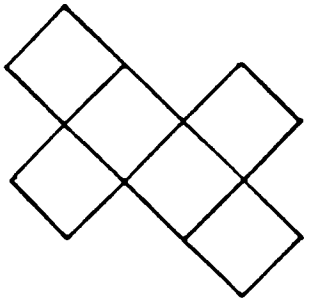
1. A= _____cm^2



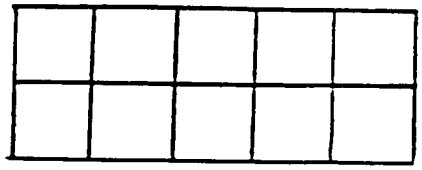
2. A= _____cm^2



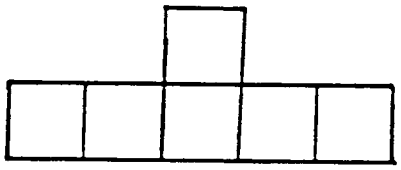
3. A= _____cm^2



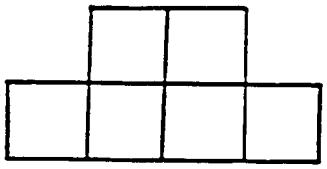
4. A= _____cm^2



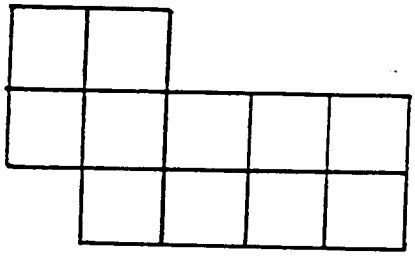
5. A= _____cm^2



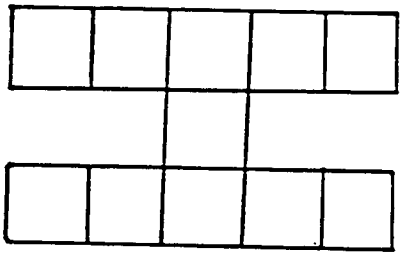
6. A= _____cm^2



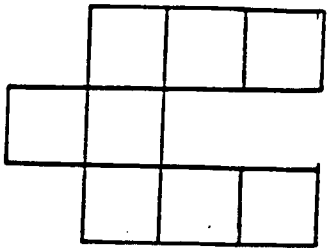
7. A= _____



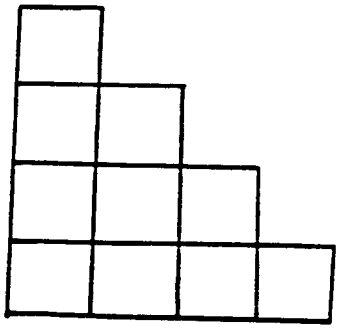
8. A= _____



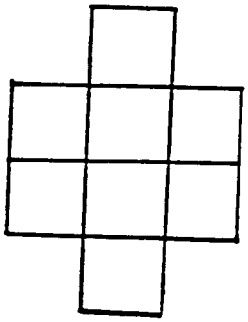
9. A= _____



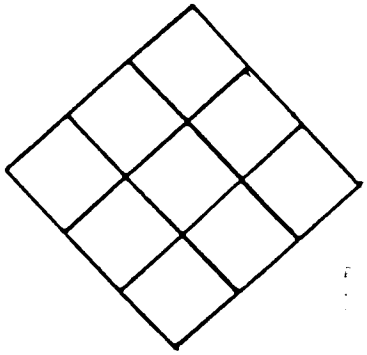
10. A= _____



11. A= _____



12. A= _____



Compass Art

- Draw a circle with a 3-inch radius.
  What is the diameter of your circle?

- Mark points on the circle using the 3-inch radius slot as a measured length.
  How many points did you mark on your line?

- Connect side-by-side points with a straight line.
  What shape did you create?

- Connect every other point with a straight line.
  What other shapes do you see in your design?

- Connect opposite points with a straight line.
  How many lines did you draw this time?

- Label intersecting points of diagonals and chords:
  A, B, C, D, E, F

- Connect: **A and C** **B and D** **C and E**
  D and F **E and A** **F and B**
  What did you add to your design that is similar to what was there before? Are they congruent? Why or why not? Are the angles the same size, larger, or smaller? How do you know?

- Use the protractor to help you make an organize list of all the angles created in your design.

- Design an emblem that you would like to use for your tile design.

adapted from Bette Kundert, Howard County Public Schools

Diving Into Pool Geometry Data Sheet

Summary of the Problem:

Information	*Formula/Algorithm*	*Measurement*
Yard 1. total perimeter of the yard	1.	1.
2. total area of the yard	2.	2.
3. final grass area left after the pool and garden are complete	3.	3.
Pool 1. total perimeter for tiling	1.	1.
2. total area for pool cover	2.	2.
3.total volume for water	3.	3.
Pool Deck 1. total perimeter	1.	1.
2. total area after pool is built	2.	2.
Garden 1. total perimeter	1.	1.
2. total area	2.	2.

Tiles - Total amount needed: _____

What mathematics knowledge did you use as you followed the steps of solving this problem Continue on the back if needed.

Student Checklist

Have we...

...clearly labeled our scale?

...checked that our grid yard measures 17 cm x 24 cm?

...checked that our pool deck measures 12 cm x 12 cm?

...checked that our pool area is a total of 60 square cm?

...checked that the area of each garden is 12 square cm?

...completed all homework and journal reflections?

...accurately calculated the formulas/algorithms on the data sheet?

...checked that our work is neat and organized?

...used math terminology in appropriate situations?

...completed the summary of the problem on the data sheet?

...completed the project reflection on the data sheet?

.....Student Checklist
Have we...

...clearly labeled our scale?

...checked that our grid yard measures 17 cm x 24 cm?

...checked that our pool deck measures 12 cm x 12 cm?

...checked that our pool area is a total of 60 square cm?

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...checked that our work is neat and organized?

...used math terminology in appropriate situations?

...completed the summary of the problem on the data sheet?

...completed the project reflection on the data sheet?

Diving Into Pool Geometry Scoring Rubric

3	Yard, pool, and garden design meets all measurement criteria. All measurements, formulas, and algorithms are accurate. Math reasoning and thinking is clear and correct. Written work is neat, organized, and utilizes proper math language.
2	Yard, pool, and garden design meet most of measurement criteria. Most measurements, formulas, and algorithms are accurate. Math reasoning and thinking is clear and mostly correct. Written work is neat, organized, and utilizes proper math language.
1	Yard, pool, and garden design meet some of the measurement criteria. Some of the measurements, formulas, and algorithms are correct. Some math reasoning and thinking are evident. Written work is legible and has some organization.
0	Yard, pool, and garden design do not meet the measurement criteria. Very few of the measurements, formulas, and algorithms are accurate. There is very little evidence of math reasoning. Written work is unorganized.